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WIND VARIABILITY AS A FUNCTION OF TIME

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ABSTRACT

Kolmogorov's structure functions for longitudinal and transverse components of isotropic turbulence when combined vectorially provide a prediction that vector time variability of the horizontal wind should vary as the cube root of the lag period. Published wind variability data are examined and found to be generally consistent with this prediction for time periods of up to 4 to 6 hr and, when representative of the hemispheric mean excluding the tropical stratosphere, may be consistent with such a prediction for periods of up to 24 to 36 hr.

1. INTRODUCTION

In a companion paper (Ellsaesser, 1969), Kolmogorov's Lagrangian structure functions for the longitudinal and transverse components of isotropic turbulence, converted to Eulerian forms by G. I. Taylor's frozen turbulence hypothesis, were combined vectorially to yield

$$(\sigma_t)^2 \equiv 2\sigma^2(1-r_t) = 7C(\epsilon\bar{u}t)^{2/3}/3 \tag{1}$$

in which σ=standard vector deviation, σ_t and r_t= Eulerian vector time variability and stretch correlation coefficient for the lag period t (all referring to the horizontal wind with a vector mean $\bar{u}$), ϵ=rate of dissipation of kinetic energy, and C is a universal constant estimated by various authors to range between ⅗ and 2. Within the approximation that $\epsilon\bar{u}$ does not vary or may be replaced by a mean value, equation (1) predicts

$$\left\{\frac{\sigma_t}{(1-r_t)^{1/2}}\right\} \propto t^{1/3}. \tag{2}$$

Validity arguments for equation (1) are offered in the companion paper (Ellsaesser, 1969). In this paper we confine our attention to time variability as a function of lag period, an investigation motivated by the conflict between (2) and the $t^{1/2}$ law represented by (6) most frequently claimed in the literature for time variability of the horizontal wind.

2. REVIEW OF $t^{1/3}$ LAW

Turbulence-type atmospheric observations consistent with the component forms of equation (2) have been reported by MacCready (1953), Taylor (1955, 1961), and Ball (1961). All found that the time variability was consistent with a $t^{1/3}$ law for equivalent distances, $\bar{u}t$, from the smallest resolvable up to a critical distance, x_c, which generally exceeded the height, z, of the observation several fold even for the vertical component (see table 1). Beyond x_c the variability of the wind components increased at a slower rate, decreased, or oscillated.

From examination of other turbulence data, Taylor (1952) had earlier proposed the $t^{1/2}$ law. This was disputed by Gifford (1956) both on theoretical grounds and from reexamination of Taylor's (1952) data. Gifford (1956) was interested in the validity of G. I. Taylor's frozen turbulenece hypothesis and deduced for three-dimensional isotropic turbulence satisfying the Kolmogorov inertial subrange $-$⅚ power law the expression

$$1-R(t)=\frac{0.788g}{1+0.891g}+0.614\left(\frac{t}{1+0.891g}\right)^{2/3},$$

$$g^2 \equiv \overline{(u-\bar{u})^2}/\bar{u}^2. \tag{3}$$

At the limits g=0 and ∞, this reduces to the ⅓ and ½ laws, respectively. Further, for the usual atmospheric range $g \leq 1$ the $R(t)$ curve lies very close to the limiting curve for g=0. Gifford (1956) interpreted this to vindicate both use of the frozen turbulence hypothesis and the $t^{1/3}$ law over the $t^{1/2}$ law for atmospheric turbulence, an interpretation with which Taylor (1957) concurred. Taylor (1961) later reexamined the observations at 29 m summarized in table 1 and additional ones at 23 and 29 m. From 31 sets of data for the u-component, he obtained for the exponent of t a mean of 0.315 with a standard deviation of 0.015.

In view of the estimates for the macroscale of turbulence (upper limit of equivalent distance, $\bar{u}t$, for equation (1), see Ellsaesser, 1969) and the values of x_c in table 1, there seems to be no reason for not extending (2) to wind variability data on the scale of standard meteorological observations of wind in the free atmosphere. The nearest approach to such an extension that has come to the author's attention is a paper by Hutchings (1955). After an extensive discussion of the applicability of turbulence theory to the free atmosphere, he concluded that the most profitable approach was to proceed with a comparison of theory and observations. In making such comparisons he found that $[1-R(t)]^{1/2}$ for the zonal and meridional components of the 6-hr 500-mb wind over Larkhill computed over 3 winter months fell nearly on a $t^{1/3}$ line for t=6 to 48 hr.

TABLE 1.—*Ratio of critical distances to height of observation at which wind variability was no longer consistent with $\sigma_t \propto t^p$ by components and values found for p.*

z (m)	$\bar{u}$ (m sec^{-1})	u		v		w		Source
		x_c/z	p	x_c/z	p	x_c/z	p	
0.075	5.7	3.9						MacCready (1953)
0.15	3.8	20						do.
2.4	5.4	1.8						do.
146	7.73	1.8						do.
1.5	2.18	5.8	0.525	5.8	0.435	2.9	0.255	Taylor (1955)
29	7.85	2.7	0.39	1.1	0.515	0.81	0.415	do.
1.5	4.47	17.7	0.27	8.8	0.36	11.8	0.095	do.
29	7.70	>10.5	0.255	1.6	0.26	1.05	0.45	do.
29	4.25	1.5	0.385	1.5	0.335	1.5	0.36	do.
1.5	3.91	15.8	0.29	10.5	0.415	<4	?	do.
29	4.60	0.48	0.42	0.63	0.32	?	?	do.
1.5	3.82	25.6	0.255	10.2	0.485	15.4	0.125	do.
7	4.18	2.4	0.305	1.8	0.385	1.5	0.34	do.

But similar data for Auckland for 300 mb for 6 to 24 hr gave exponents of 0.44.

Most studies of wind variability have not been processed in a manner that permits testing any of the deductions of similarity theory other than equation (2), and the law most frequently quoted is not $t^{1/3}$ but $t^{1/2}$. In view of the observational studies quoted above, it seems strange that a larger observing scale and larger values of t should result in a more rapid rate of increase of the structure function. A possible rationalization of this apparent anomaly is that similarity theory is derived in wave number space, i.e., geographical inhomogeneities are averaged out. In physical space, energy released on a particular scale could be concentrated in specific areas such as the well-recognized major cyclone storm tracks of the westerlies. In such areas, local increases in turbulent energy with scale, at rates faster than predicted by similarity theory, become comprehensible. Similarly, increases at slower rates should be anticipated in regions such as the lower tropical troposphere that are relatively isolated from perturbations of the so-called synoptic scale.

3. REVIEW OF $t^{1/2}$ LAW

The $t^{1/2}$ law represented by equation (6) apparently originated from the early statistical theory of turbulence prediction that two particles should separate at a rate proportional to the square root of their distance apart (Arnold, 1956). Nowhere in the literature on wind variability did the author find an appeal to the Lagrangian form of equation (1) as support for the $t^{1/2}$ law.

For time variability, Durst (1954) used the relation from (1)

$$\sigma_t = \sigma(2-2\mathrm{r}_t)^{1/2} \tag{4}$$

and empirically fitted the power law

$$\mathrm{r}_t = e^{-\alpha t}. \tag{5}$$

For British data, he determined $\alpha = 2.484 \times 10^{-2}$ hr^{-1}. Evaluation of α from other data samples shows that it is more stable than σ or σ_t but does have significant geographical and seasonal variations. Also equation (5) does not appear to hold very well for time periods in excess of 36 hr (Ellsaesser, 1960).

Reed (1967) substituted the series expansion of equation (5) into (4) to show that

$$\sigma_t \propto t^{1/2} \tag{6}$$

in agreement with previous empirical findings (Reed, 1958), provided $e^{-\alpha t}$ can be approximated by $1-\alpha t$. Using Durst's value (1954) of α, (6) reproduces equations (4) and (5) with a 1-percent accuracy for $t<6$ hr; 10-percent for $t<18$ hr.

Eriksson (1961) integrated Taylor's (1921) formula for the variance, X^2, of x-distances traversed by air particles in an x-velocity field of variance U^2 and Lagrangian time correlation, R_t,

$$X^2 = 2U^2 \int_0^T \int_0^t R_\xi \, d\xi \, dt, \tag{7}$$

under the assumptions: a) T is so short $R_t \approx 1$, and b) R_t drops to zero before $t=T$ but the ξ-integral is finite. He then assumed the x-distances over T were equivalent to the spreading of balloons in a series of ascents and that the latter are measured by the standard vector deviation of the differences between consecutive wind vectors, i.e., vector time variability of the wind. He thus arrived at

$$\sigma_t = \sigma_1 t^p, \qquad \tfrac{1}{2} \leq p \leq 1 \tag{8}$$

with $p=\tfrac{1}{2}$ valid for "long" time periods which he assumed to be periods of 1 hr or more. Using samples of summertime wind variability data from southern Sweden for lag periods of 1 to 6 hr, he empirically determined a mean value of $p=0.51$.

Eriksson did not take the next step, apparent from equation (4), that after r_t goes to zero

$$\sigma_t = \sqrt{2}\sigma, \tag{9}$$

i.e., $p=0$. This should have caused him to doubt his use of equation (7). The essence of T in (7) is to denote a time period over which a trajectory (or wind) is averaged. The formula then relates, through the Lagrangian correlation, the dispersion of these averaged trajectories to the standard deviation of the velocity field. Thus one may object to Eriksson's use of (7) because: a) he allows the averaging time T to far exceed the usual sampling time of at most a few minutes for measuring upper level winds; b) at no point does (7) introduce the concept of a definite interval between successive trajectories (wind vectors); and c) for meteoroligical observations of winds aloft $R_t > r_t$ and the latter does not in general vanish for time periods of less than several days (Charles, 1959*a*).

Eriksson (1961) corrected his data for observational error by subtracting $2(\sigma_e)^2$ from the measured time variance of the wind. In some cases this prevented him from obtaining variability data for periods shorter than 3 hr due to negative residual suggesting that his σ_e was

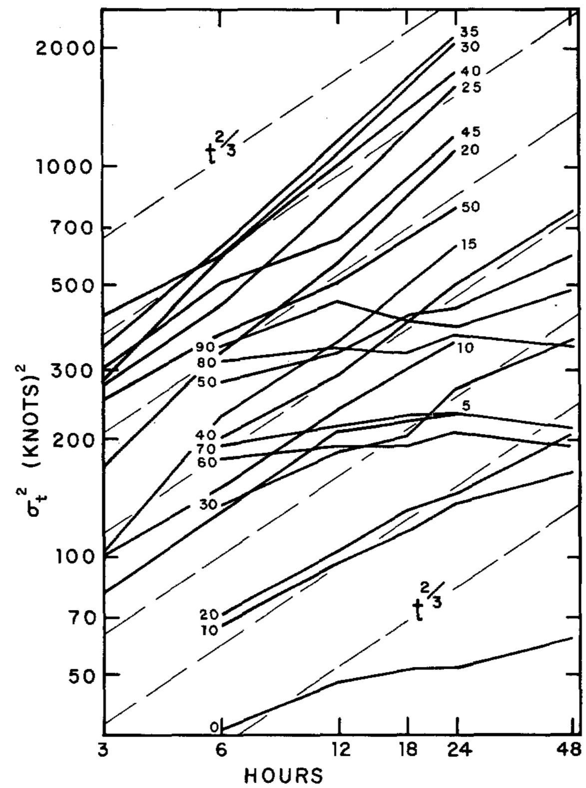

FIGURE 1.—Log log plot of time variance, $(\sigma_t)^2$, of the horizontal wind as a function of time. Altitudes in thousands of feet of spring Nevada Test Site data (Reed, 1958) are entered along 24-hr line, of annual average Eniwetok data (Reed, 1967) along 6-hr line. Slope of $t^{2/3}$ ($t^{1/3}$ law) is indicated by dashed lines, slope of t^1 ($t^{1/2}$ law) is one (45°).

too large. If so, this would exaggerate the apparent rate of increase of σ_t with t. Such an effect is suggested by the general decrease of p in equation (8) with increases in the average wind speed of his samples from 0.67 to 0.50 for winds <5 m sec^{-1} to 0.37 for winds >15 m sec^{-1}. The latter value should be the least affected and is very near the 0.33 predicted by equation (2). This suggests that Eriksson's (1961) data may be consistent with the $t^{1/3}$ law for lag periods of 1 to 6 hr.

Reed's $t^{1/2}$ law (1958, 1967) is apparently based on extensive empirical results. However, only limited data samples are included in his reports. To give a maximum separation of slope, the NTS (Nevada Test Site) data for spring (Reed, 1958) and the Eniwetok data (Reed, 1967) are plotted on log paper as $(\sigma_t)^2$ versus t in figure 1. In this figure the $t^{1/2}$ law should appear as a straight line with a slope of one (45° angle) and the $t^{1/3}$ law as a straight line paralleling the dashed lines in the figure. Only the NTS data for 15,000 and 30,000 ft and only between 3 and 6 hr actually reached or exceeded the $t^{1/2}$ law. However, the 20,000- to 40,000-ft data increased almost as rapidly as required by $t^{1/2}$ for 3 to 24 hr. Since σ_t, $\bar{u}$, and ϵ undoubtedly vary together and these levels over NTS frequently include the jet stream, it is believed that these data would still provide a reasonable fit to equation (1) if the variations of $\bar{u}$ and ϵ were considered.

Reed (1967) did not claim that the Eniwetok variability data supported the $t^{1/2}$ law, but since they were included in a report again espousing this law, they were included in figure 1. Only at 10,000–40,000 ft do they approach even the $t^{1/3}$ law. At the surface, the shape of the curve suggests that the macroscale of tubulence is exceeded at 12 hr invalidating equation (1). This might also be the interpretation at levels above 40,000 ft. Another possible explanation at these upper levels is the magnitude of errors of observation relative to the time wind variability. Neither is considered valid for this unique wind regime of the tropical stratosphere first revealed by the trajectory of the dust cloud from the Krakatoa eruption of 1883. Palmer et al. (1955) call the steadiness of these winds astonishing and point out that their steadiness is a direct function of the zonal speed. Charles (1959*b*) found that the time-lag correlation of the zonal wind component at 50 and 30 mb over Panama had decreased to only 0.48 and 0.69 after 10 days. Reed (1967) shows that the lag correlation of the meridional component over Eniwetok from 60,000 to 90,000 ft is essentially zero at 12 hr while for the zonal component it ranges from 0.346 to 0.782 at 48 hr. This difference does not appear explainable in terms of observational error. The one thing that does appear clear is that the tropical stratosphere is not a region of isotropic tubulence on a time scale exceeding 6 hr.

In an effort to obtain a more definitive answer, variability data from Durst (1954), Singer (1956), and Plagge and Smith (1956) were plotted in figure 2 (same scale as fig. 1) and the plot rotated 45° so that t^1 ($t^{1/2}$ law) would appear as a vertical line; $t^{2/3}$ ($t^{1/3}$ law) lines are the light dashed lines sloping upward and to the right. Singer's (1956) data, from 226 triple theodolite soundings in 89 observing days spread over a full year at Muroc, Calif., and separated into four 10,000-ft altitude zones, appear at the lower right. Unfortunately, his data are not homogeneous since different lag periods are averages of different sets of soundings. Also, he combined different lag periods and plotted points as function of the average period—a valid procedure only if σ_t is a linear function of t. Even after making allowances for these deficiencies, Singer's (1956) data display an unexplained anomalously rapid increase of wind variability with lag period compared to other data. At this point they can only be regarded as unrepresentative.

Plagge and Smith's (1956) data from 530 balloon releases near the Salton Sea tracked by SCR–584 radar and GMD–1A rawin on 12 observing days in spring appear as the lines beginning in the lower left part of figure 2 and terminating with a check mark profile joining the 9- and 12-hr points. Only data for the eight altitude zones below 40,000 ft were plotted. These data also are not homogeneous in that the number of cases varies with lag period, dropping significantly for lags of 7, 8, and 9 hr. This is considered to be the cause of both the decrease in variability with time for short lag intervals and of the

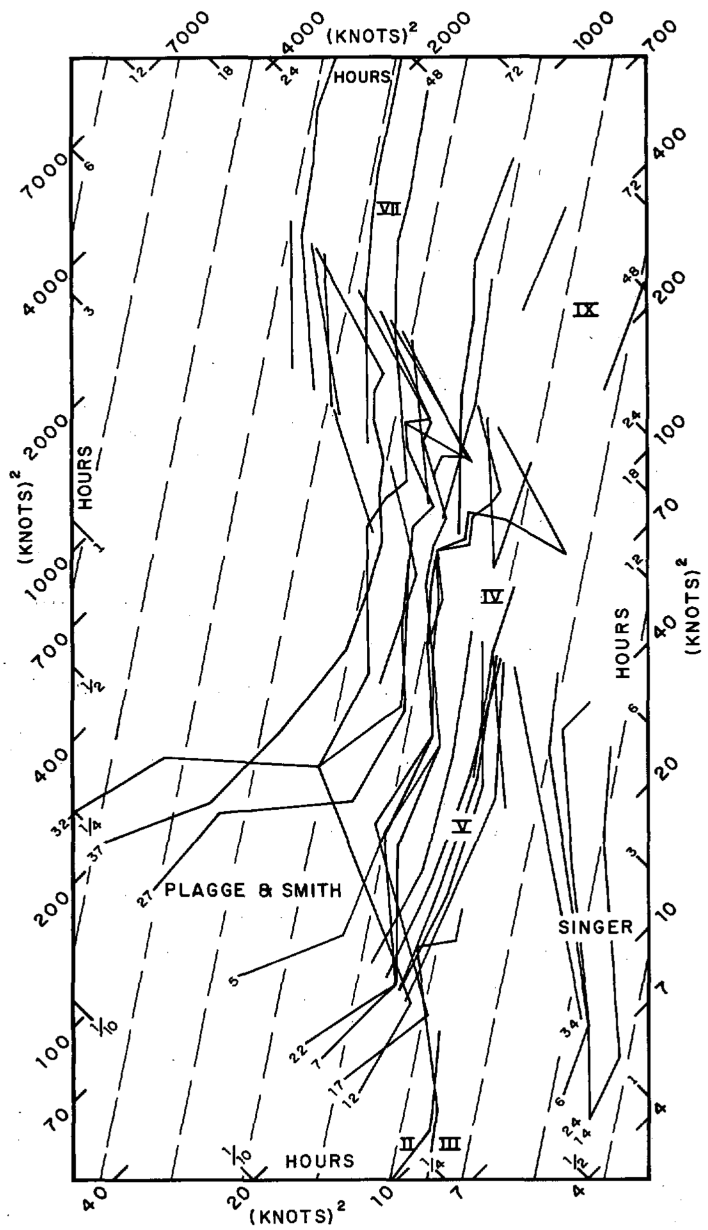

FIGURE 2.—Same as figure 1 using data from Durst (1954), Singer (1956), and Plagge and Smith (1956). Chart rotated 45° to left so that t^1 ($t^{1/2}$ law) lines are vertical. Slope of $t^{2/3}$ ($t^{1/3}$ law) indicated by the dashed lines. Arabic numerals are midpoints of altitude zones of samples in thousands of feet. Roman numerals are source table numbers in Durst (1954).

rapid increase between 9 and 12 hr. While the observing errors were appreciable, σ_e estimated at 3 to 4.6 kt for all levels combined and 3.5 to 6.7 kt for levels above 25,000 ft, their effect seems to be masked by the lack of homogeneity. If the slopes between 1 and 6 hr can be considered representative, these data are consistent with a $t^{1/3}$ law.

Durst (1954) provides the most complete set of reasonably homogeneous data. His various samples from the British Isles span lag periods of one-tenth to 48 hr with high observing accuracy for the shorter periods. The Roman numerals in figure 2 are his table numbers from which the data were taken. For lag periods up to 4 hr, his data are reasonably consistent with a $t^{1/3}$ as opposed to a $t^{1/2}$ law. On the other hand, his Larkhill data (his table VII) display something closer to a $t^{1/2}$ law between 6 and 24 hr.

Durst (1954) also presents space variability data computed from vector differences between 3,000-ft winds at Shoeburyness and various nearby stations ranging as far away as Berlin. The variability increased slightly faster than the square root of the distance, s. Arnold (1956) linked the data with his own and used them to support the $s^{1/2}$ law, as he reported Durst had done earlier. Arnold's points at one-half and 5 km in New Jersey indicate an $s^{1/4}$ law, and connecting them with Durst's (1954) nearest point for 110 km computed over Britain is unlikely to give a meaningful estimate of the distance exponent. Since Durst's (1954) values are vector differences in wind and make no allowance for the difference in mean vector winds for the two end points (these differ appreciably between Southern England and Berlin), they should not be expected to obey turbulence theory. Two stations with winds constant at their mean vector values for the period could give comparable values of σ_s when computed in this way. That Durst himself was not satisfied with these data is suggested by his omission of the resultant-estimated space correlations from the graphs in his figure 6.

4. CONCLUSIONS

Because measurements of time variability of the wind are made locally in configuration space, its variation with time need follow no fixed prescription. If the observations are sufficiently (but not too remotely) removed from atmospheric energy sources either through restrictions on the scale of the measurements or through the geographical location of the observation points with respect to the major storm tracks or both; or if the effects of energy sources are smoothed out by averaging data from both active and inactive regions, then variation in accordance with the predictions of turbulence theory becomes a reasonable expectation.

From Gifford's (1956) theoretical results and from the empirical data examined in this report, it is concluded that for lag periods up to 4 to 6 hr time variability of meteorologically observed winds is more nearly consistent with the $t^{1/3}$ law than with the $t^{1/2}$ law. It is also believed that the $t^{1/3}$ law applies reasonably well to a hemispheric average wind variability for lag periods of 6 to 36 hr since stations near the westerly storm belt show a faster and those in the lower tropical troposphere show a slower rate of increase of variability with lag period in this range.

Measurements of time variability of the wind should be expected to depart from the $t^{1/3}$ law under any of the conditions listed below. Whether there are additional exceptions to the $t^{1/3}$ law remains to be determined.

a) Over short lag periods when the standard error of observation exceeds one-third of the measured time variability; this represents an error in the measured variability exceeding 10 percent.

b) In nonhomogeneous samples; i.e., different lag periods are composed of other than overlapping observing periods at a single site.

c) When the sample includes significant variations in wind speed and dissipation so that use of a mean value for $\epsilon\overline{u}$ is inappropriate.

d) For time periods in excess of 6 hr at stations either 1) near a major storm track in the westerlies as represented by Larkhill where appreciable kinetic energy is being generated locally by baroclinic cyclogenesis or 2) isolated from the major storm tracks as in the lower tropical troposphere represented by Eniwetok.

e) For time periods in excess of 6 hr in the tropical stratosphere where turbulence on this scale is definitely not isotropic.

This study was undertaken in an effort to test the validity of applying similarity theory as represented by equation (1) to wind variability data as a means of estimating dissipation, ϵ, in the free atmosphere. The author was sufficiently encouraged to proceed with such an evaluation in the companion paper (Ellsaesser, 1969).

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